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Bifurcation Analysis of Nonlinear Periodic Systems via Lyapunov - Floquet Technique			
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13. ABSTRACT (Maximum 200 words)			
A technique has been presented for computing the Lyapunov-Floquet transformation matrix for multidimensional nonlinear systems with periodically varying parameters by first computing the state transition matrix (STM) of the linearized system in a symbolic form via Chebyshev polynomials over the principal period. This STM is then factored to evaluate the Lyapunov-Floquet transformation matrix which is used to transform the nonlinear periodic system to one with time invariant linear coefficients. The subsequent application of center manifold and normal form theories may then be used to determine the stability and obtain an accurate analytical solution which is suitable for algebraic manipulations. Unlike the averaging and perturbation techniques, the proposed technique does not require the existence of a small parameter multiplying the time varying terms. Application of this method to externally excited systems such as periodically loaded columns and rotordynamic systems has given an accurate representation of the resonance conditions which arise in the system. Also, a method for obtaining the STM of the linearized system with parametric dependence has been shown. When combined with the stability and bifurcation theory of discrete maps, accurate local bifurcation surfaces may be obtained in closed form in the parameter space. Results indicate that this technique is well converged and is computationally more efficient than comparable schemes such as point mapping.			
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**TITLE: BIFURCATION ANALYSIS OF NONLINEAR PERIODIC SYSTEMS  
VIA LYAPUNOV-FLOQUET TECHNIQUE**

(Contract Period: September 1, 1992 - August 31, 1996)

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## Accomplishments

A technique has been presented for computing the Lyapunov-Floquet transformation matrix for multidimensional linear systems with periodically varying parameters. In this technique, the state vector and the periodic linear coefficient matrix are expanded in Chebyshev polynomials over the principal period. Such an expansion reduces the original differential problem to a set of linear algebraic equations. From the solution of the resulting algebraic system, the state transition matrix (STM) of the linear equations can be computed in a symbolic form. This STM is then factored to evaluate the Lyapunov-Floquet transformation matrix which is used to transform the nonlinear periodic system to one with time invariant linear coefficients. The subsequent application of center manifold and normal form theories may then be used to determine the stability and obtain a solution. The solutions thus obtained using this method compare very well with the Runge-Kutta type numerical integration results. However, unlike the numerical results, the solutions obtained using this method are expressed in analytical forms which are suitable for algebraic manipulations. Unlike the averaging and perturbation techniques, the proposed technique does not require the existence of a small parameter multiplying the time varying terms. Also, the symbolic inversion of large matrices is avoided by using the adjoint form of the original system of equations. Application of this method to externally excited systems such as periodically loaded columns and rotordynamic systems has given an accurate representation of the resonance conditions which arise in the system.

New methods for use of the Chebyshev Operational Matrices to solve systems of differential equations with arbitrary order (and subsequently construct the Lyapunov-Floquet transformation matrix and its inverse) using “differential” and “hybrid” formulations has been studied. Also, a valuable discussion on the theoretical justifications and advantages for solving

time-periodic systems via Chebyshev polynomials has been made where it was observed that the approach is several times faster in terms of CPU time than (and with accuracy comparable to) higher order numerical algorithms such as Runge-Kutta, Adams-Molton, and Gear methods.

A generalized procedure for studying the bifurcation analysis of Hamiltonian systems using the Liapunov-Floquet transformation to convert the quadratic portion of the Hamiltonian function from a time periodic form to a time invariant one has been outlined. It was found that the transformation may be used in conjunction with the use of permutation matrices as well as the well-known methods of canonical perturbation theory to obtain a solution to the original time-periodic Hamilton's equations.

Finally, a method for obtaining the Floquet Transition Matrix (FTM) of the linearized system with parametric dependence has been shown. When combined with the stability and bifurcation theory of discrete maps, accurate local bifurcation surfaces may be obtained in closed form in the parameter space. In performing these computations, symbolic software such as MATHEMATICA is required. Two formulations for systems of arbitrary dimension have been outlined and convergence and CPU times have been discussed in detail. The results indicate that this technique is well converged, does not require the use of small periodic terms as in averaging and perturbation, and is computationally more efficient than comparable schemes such as point mapping. Examples include a Mathieu equation as well as a double inverted pendulum subjected to a periodically varying follower force.

## LIST OF PUBLICATIONS

### Archival Publications:

#### a) Publications already in print (copies attached):

1. Sinha, S. C., R. Pandiyan, and J. S. Bibb, "Liapunov-Floquet transformation: computation and application to periodic systems," *J. Vibration and Acoustics* 118, 209-219 (1996).
2. Sinha, S. C. and Eric A. Butcher, "Solution and stability of a set of  $p$ th order linear differential equations with periodic coefficients via Chebyshev polynomials," *Mathematical Problems in Engineering*, 2, 165 (1996).
3. Sinha, S. C. and E. A. Butcher, Author's reply to "Some observations on the Sinha approach to dynamic response calculations" by D. A. Peters, *J. Sound and Vibration* 188(4), 629(1995).
4. Butcher, Eric A. and S. C. Sinha, "A Hybrid Formulation for the Analysis of Time-Periodic Linear Systems via Chebyshev Polynomials," *J. Sound and Vibration* 195(3), 518 (1996).

#### b) Submitted for Publication:

5. Sinha, S. C. and Eric A. Butcher, "Symbolic approximation of the fundamental solution matrix for a time-periodic dynamical system," submitted to *J. Sound and Vibration*.

### Proceedings and Presentations at Professional Conferences:

6. Sinha, S. C., R. Pandiyan, and D. M. Gibas, "Dynamic analysis of rotor-bearing systems via Chebyshev polynomials," *Structural Dynamics and Vibrations*, proceedings of the 1994 ASME Engineering Systems Design and Analysis Conference, PD v. 64-7, July 4-7, 1994, London, U. K. ed. B. Ovunc, *et. al.*, pp. 187-194.
7. Pandiyan, R., E. A. Butcher, and S. C. Sinha, "On the approximation of dynamical systems subjected to strong parametric excitations," *Nonlinear and Stochastic Dynamics*, 1994 Winter Annual ASME Meeting proceedings, Nov. 6-11, 1994, Chicago, IL, AMD v. 192, DE v. 78, ed. A. K. Bajaj, N. S. Namachchivaya, and R. A. Ibrahim, pp. 275-283.
8. Butcher, Eric A. and S. C. Sinha, "On the analysis of time-periodic nonlinear Hamiltonian dynamical systems," *Vibration of Nonlinear, Random, and Time-Varying Systems*, proceedings of the 1995 Design Engineering Technical Conferences, v. 3-A, Sept. 17-21, 1995, Boston, MA, DE v. 84-1, ed. S. C. Sinha, *et. al.*, pp. 375-386.

9. Butcher, Eric A. and S. C. Sinha, "Analysis of Nonlinear Time-Periodic Hamiltonian Systems," presented at the 32nd Annual Technical Meeting of the Society of Engineering Science, Oct. 29 - Nov. 2, 1995, New Orleans, LA, proceedings p. 73.
10. Sinha, S. C. and Eric A. Butcher, "Symbolic Computation of Stability and Bifurcation Surfaces for Nonlinear Systems with Strong Parametric Excitation," presented at the 6th Nonlinear Vibrations, Stability, and Dynamics of Structures Conference, June 9-13, 1996, Virginia Polytechnic Institute and State University, Blacksburg, VA.
11. Butcher, Eric A., "Symbolic Approximation of Stability Boundaries for Strongly Excited Time-Periodic Systems," presented at the ASME Southeast Region XI Graduate Student Technical Conference, May 17-18, 1996, University of Alabama, Tuscaloosa, AL.

### **STUDENTS SUPPORTED**

<b>Name</b>	<b>Period of Support</b>	<b>Degree Awarded(Year)</b>	<b>Citizenship</b>
John S. Bibb	9/92 - 3/93	M.S. (1992)	U.S. Citizen
Darrell Gibas	3/93 - 12/93	None	U.S. Citizen
Eric A. Butcher	1/94 - 8/96	M.S. (1995) (Ph.D. near completion)	U.S. Citizen